A New and Feasible Protocol for Semi-quantum Key Distribution

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Abstract. Quantum Key Distribution (QKD) protocols make it possible for two quantum parties to generate a secret shared key. Semi-quantum Key Distribution (SQKD) protocols, such as "QKD with classical Bob" and "QKD with classical Alice" (that have both been proven robust), achieve this goal even if one of the parties is classical. However, the currently existing SQKD protocols are not experimentally feasible with the current technology. Here we suggest a new protocol ("Classical Alice with a controllable mirror") that can be experimentally implemented with the current technology, and prove it to be robust.

1   Introduction

Quantum Key Distribution (QKD) makes it possible for two legitimate parties, Alice and Bob, to generate an information-theoretically secure key [1], that is secure against any possible attack allowed by the laws of quantum physics. Alice and Bob use an insecure quantum channel and an authenticated classical channel. The adversary Eve may interfere with the quantum channel and is limited only by the laws of nature; she may not, however, modify the data sent in the authenticated classical channel (she can only listen to it).

Semi-quantum Key Distribution (SQKD) protocols limit one of the parties to be classical, yet giving a secure key [4]. The first SQKD protocol was "QKD with classical Bob" [4]; later, the "QKD with classical Alice" [22,8] protocol was suggested, as well as various other SQKD protocols (see for example [12,19,21]). Most of the SQKD protocols have been proven "robust": namely [4], any successful attack by an adversary necessarily induces some noise that the legitimate parties may notice (see also the formal definition in Sect. 2.3). A few of them have also been proven secure [11].

However, to the best of our knowledge, all the currently existing SQKD protocols cannot be experimentally constructed in a secure way by using the current technology, as explained in Sect. 1.3. In other words, despite the fact that SQKD protocols should have been easier to implement than QKD protocols (because only one party requires quantum abilities), it turns out that some of the "classical" operations are very hard to implement in a secure way.

We present a new Semi-quantum Key Distribution protocol that can be experimentally constructed by using a "controllable mirror". It is based on "QKD with classical Alice" [22,8], but it is more complicated, because it allows Alice to choose one of four operations (instead of two). We prove this protocol to be robust.

1.1   Quantum Key Distribution Protocols

QKD protocols achieve the classically-impossible goal of distributing a secret key to two parties (Alice and Bob), in a way that is secure against all the possible attacks. Moreover, the key shared by Alice and Bob remains secret even if weaknesses in the devices (currently unknown to anyone, *including* the adversary) are discovered in the future: namely, for the adversary Eve to find the key, she must attack when Alice and Bob apply the protocol, and not later (while for encryption methods such as RSA, Eve may keep the ciphertext until she is able to find the private key, e.g., by factorizing a large number).

The first QKD protocol was BB84 [1]:

Protocol 1 (BB84). *The BB84 protocol, operated by the two parties Alice and Bob, consists of the following steps:*

1. *Alice sends to Bob N quantum states, all of them randomly chosen from the following set:* $\{|0\rangle, |1\rangle, |+\rangle \triangleq \frac{|0\rangle+|1\rangle}{\sqrt{2}}, |-\rangle \triangleq \frac{|0\rangle-|1\rangle}{\sqrt{2}}\}$
2. *Bob measures all the received states; for each of the states, he chooses randomly whether to measure it in the computational basis* $\{|0\rangle, |1\rangle\}$ *or in the Hadamard*

basis $\{|+\rangle, |-\rangle\}$. *If Bob measures in the computational basis, he identifies* $|0\rangle$ *and* $|1\rangle$ *with certainty, but gets a random result if* $|+\rangle$ *or* $|-\rangle$ *is sent; the converse is true for the Hadamard basis.*

3. *Now Alice and Bob each holds a (classical) bit string: Alice holds the list of bits she sent (bit* 0 *corresponding to the states* $|0\rangle$ *and* $|+\rangle$, *and bit* 1 *corresponding to the states* $|1\rangle$ *and* $|-\rangle$*), and Bob holds the list of bits he measured (with similar interpretations as Alice). In addition, Alice knows the basis she used to send each state, and Bob knows the basis he used to measure each state.*

4. *Alice and Bob reveal (by using the classical channel) their basis choices, and discard all the states that Bob measured in a basis different from the one sent by Alice.*

5. *Alice and Bob reveal some random subset of their bit string, compare the bits, and estimate the error rate. They abort the protocol if the error rate is above a specified threshold (in BB84, the asymptotic threshold (for infinite key-length) is 11% [17,18]). They discard the revealed bits.*

6. *Now Alice and Bob keep only the string of bits that were measured by Bob in the same basis they were sent by Alice (and that were not discarded). If there is no noise or eavesdropping, this bit string should be the same for Alice and Bob.*

7. *Alice sends to Bob error correction information, and Bob corrects the errors in his bit string, so that it is the same as Alice's.*

8. *Alice and Bob perform a privacy amplification process, yielding a final key that is identical and is fully secure from any eavesdropper.*

The notion of "(composable) full security" of a protocol (informally) means that, except with an exponentially-small probability ε, one of the two following events happens: the protocol is aborted, *or* the secret key generated by the protocol is the same as a perfect key that is uniformly distributed, is the same for both parties, and is independent of the adversary's information [17,16] (see earlier definitions of "full security", that were not composable, in [14,2,18]).

Many QKD protocols have been proven fully (and unconditionally) secure in the theoretical sense [17,16].

However, practical implementations deviate from the theoretical descriptions, and may thus be insecure. Two important attacks that take advantage of this fact are the "Photon Number Splitting" attack [10,9] and the "Bright Illumination" attack [13]. The "Photon Number Splitting" attack takes advantage of the fact that Alice cannot generate only one-photon pulses, but sometimes generates pulses of two (or more) photons: Eve can, under certain conditions, get full information on the secret key without inducing errors. The "Bright Illumination" attack uses a weakness of Bob's detectors, existing in some practical implementations, to get full information on the secret key without inducing errors.

Other QKD protocols, either similar to BB84 or ones that use different approaches, have also been suggested, and in some cases have also been proven fully secure.

1.2 Existing Semi-quantum Key Distribution Protocols

The notion of a "Semi-quantum Key Distribution" (SQKD) protocol, in which one of the parties uses only classical operations, was introduced in [4,5].

For the purpose of the definition of SQKD protocols, the term "classical operations" refers to the following operations [3]:

1. Measuring a state in the computational basis $\{|0\rangle, |1\rangle\}$.
2. Generating a state in the computational basis $\{|0\rangle, |1\rangle\}$ and sending it to the other party.
3. Reordering quantum states, without measurement. (This operation is not used in this paper, but is useful for randomization-based protocols, such as the protocols described in [5,3].)
4. Moving or reflecting a quantum system or subsystem, without changing it.
5. Choosing a random bit.
6. All the regular operations on the classical channel, classical computation, etc.

The operations are "classical" in the sense that they only treat the computational basis $\{|0\rangle, |1\rangle\}$ (or do nothing). If <u>both</u> Alice and Bob are classical in that sense, and the only interaction is between them, then the protocol is classical, and thus secure key distribution is impossible; however, if one party is classical and the other is quantum, the key distribution protocol may be secure.

Quantum Key Distribution with Classical Bob. The SQKD protocol "Quantum Key Distribution with classical Bob" [4] is defined as follows:

Definition 2 (Classical Operations in "QKD with Classical Bob"). *The classical operations of Bob in the "QKD with classical Bob" protocol are:*

CTRL *Return the received state to Alice, without modifying or measuring it. (Here, Bob uses the classical operation 4.)*
SIFT (measure+resend) *Measure the received state in the computational basis $\{|0\rangle, |1\rangle\}$, prepare a new state ($|0\rangle$ or $|1\rangle$) according to the measured bit, and send the new state back to Alice. (Here, Bob uses the classical operations 1+2.)*

Protocol 3 ("QKD with Classical Bob"). *The "QKD with classical Bob" protocol (using a two-way quantum channel), operated by the two parties Alice and Bob (where Bob is classical, in the sense defined above), consists of the following steps:*

1. *Alice sends to Bob N quantum states, all of them randomly chosen from the following set: $\{|0\rangle, |1\rangle, |+\rangle \triangleq \frac{|0\rangle+|1\rangle}{\sqrt{2}}, |-\rangle \triangleq \frac{|0\rangle-|1\rangle}{\sqrt{2}}\}$*
2. *For each of the received states, Bob randomly chooses (by using the classical operation 5) one of the two classical operations listed in Definition 2 (CTRL or SIFT).*
3. *Alice measures all the states she receives in the same basis she sent them.*
4. *Now Alice and Bob each holds a (classical) bit string: Alice holds the list of bits she sent (bit 0 corresponding to the states $|0\rangle$ and $|+\rangle$, and bit 1 corresponding to the states $|1\rangle$ and $|-\rangle$), and Bob holds the list of bits he measured when he used SIFT. In addition, Alice knows the basis she used to send each state; Bob knows the operation he used for each state (SIFT or CTRL); and Alice holds a bit string representing her final measurements, that will be used later for checking the error rate.*

5. *Alice and Bob reveal (by using the classical channel) Alice's basis choices and Bob's operation choices.*
6. *Alice checks the error rate in the CTRL bits.*
7. *Alice and Bob reveal some random subset of the SIFT bits sent by Alice in the computational basis, compare them, and estimate the error rate (both in the way from Alice to Bob and in the way from Bob back to Alice). They abort the protocol if the error rate in those SIFT bits or in the CTRL bits is above a specified threshold. They discard the revealed bits.*
8. *Now Alice and Bob keep only the string of SIFT bits that were sent by Alice in the computational basis (and that were not discarded). If there is no noise or eavesdropping, this bit string should be the same for Alice and Bob.*
9. *Alice sends to Bob error correction information, and Bob corrects the errors in his bit string, so that it is the same as Alice's.*
10. *Alice and Bob perform a privacy amplification process, yielding a final key that is identical and is fully secure from any eavesdropper.*

This protocol was proved to be robust in [4], and was later proved to be secure [11].

Quantum Key Distribution with Classical Alice. An extension to "QKD with classical Bob", in which the originator always sends the same state $|+\rangle$, was suggested in [22]. Following [8], we prefer to call the originator in [22] Bob (and not Alice), and to call the classical party Alice. Thus, we call the SQKD protocol of [22] "QKD with classical Alice".

Definition 4 (Classical Operations in "QKD with Classical Alice"). *The classical operations of Alice in the "QKD with classical Alice" protocol are:*

CTRL *Return the received state to Bob, without modifying or measuring it.*
SIFT (measure+resend) *Measure the received state in the computational basis $\{|0\rangle, |1\rangle\}$, prepare a new state ($|0\rangle$ or $|1\rangle$) according to the measured bit, and send the new state back to Bob.*

Protocol 5 ("QKD with Classical Alice"). *The "QKD with classical Alice" protocol (using a two-way quantum channel), operated by the two parties Alice and Bob (where Alice is classical, in the sense defined above), consists of the following steps:*

1. *Bob sends to Alice N quantum states, all equal to $|+\rangle \triangleq \frac{|0\rangle + |1\rangle}{\sqrt{2}}$.*
2. *For each of the received states, Alice randomly chooses one of the two classical operations listed in Definition 4 (CTRL or SIFT).*
3. *Bob measures all the states he receives, choosing randomly for each state whether to measure it in the computational basis $\{|0\rangle, |1\rangle\}$ or in the Hadamard basis $\{|+\rangle, |-\rangle\}$.*
4. *Now Alice and Bob each holds a (classical) bit string: Alice holds the list of bits she measured when she used SIFT (bit 0 corresponding to the state $|0\rangle$, and bit 1 corresponding to the state $|1\rangle$), and Bob holds the list of bits he measured. In addition, Alice knows the operation she used for each state (SIFT or CTRL), and Bob knows the basis he used to measure each state.*

5. *Alice and Bob reveal (by using the classical channel) Alice's operation choices and Bob's basis choices.*
6. *Bob checks the error rate in the CTRL bits that he measured in the Hadamard basis.*
7. *Alice and Bob reveal some random subset of the SIFT bits measured by Bob in the computational basis, compare them, and estimate the error rate (in the way from Alice back to Bob). They abort the protocol if the error rate in those SIFT bits or in the CTRL bits is above a specified threshold. They discard the revealed bits.*
8. *Now Alice and Bob keep only the string of SIFT bits that were measured by Bob in the computational basis (and that were not discarded). If there is no noise or eavesdropping, this bit string should be the same for Alice and Bob.*
9. *Alice sends to Bob error correction information, and Bob corrects the errors in his bit string, so that it is the same as Alice's.*
10. *Alice and Bob perform a privacy amplification process, yielding a final key that is identical and is fully secure from any eavesdropper.*

Note that a different protocol (also named "Classical Alice") was suggested in [12], independently of [22].

As proven in [8], "QKD with classical Alice" [22] is completely robust against eavesdropping. The proof of robustness was extended in [7] to include photonic implementations and multi-photon pulses.

1.3   The Experimental Infeasibility of the SIFT Operation in SQKD Protocols

In many SQKD protocols (such as "QKD with classical Bob" and "QKD with classical Alice" described above), it is assumed that the classical party can either do nothing (the CTRL operation) or measure in the computational basis $\{|0\rangle, |1\rangle\}$ and then resend (the SIFT operation). In practical (photonic) implementations, and especially if limited to the existing technology, it is quite impossible for the classical party to do that, and the photon generated by him or her during the SIFT operation will probably be at a different timing or frequency, thus leaking information to the eavesdropper; see comment on [4] and the reply [20,6].

For example, let us look at the "QKD with classical Alice" protocol, and assume that the two classical states, $|0\rangle$ and $|1\rangle$, describe two pulses (in two distinct time-bins) on the same arm, such that the photon can either be in one pulse, in the other, or in a superposition (that is a non-classical state).

Given that implementation, it is indeed very difficult for Alice to regenerate the SIFT photons at the right timing. Furthermore, in [20] it is shown that even if Alice could have the machinery to SIFT with perfect timing, Eve can make use of the fact that Alice does not detect the state with perfect qubit-detectors: Eve can modify the frequency of the photon generated by Bob. Then, if Alice SIFTs, she generates a photon in the original frequency, while if she performs the CTRL operation, the reflected photon is in the frequency modified by Eve. Then, by measuring the frequency, Eve can tell whether Alice used the SIFT or the CTRL operation; if Eve finds out that Alice used CTRL, she shifts the frequency back to the original frequency, while if she finds out that Alice used SIFT, she can copy the bit sent by Alice in the computational basis. This "tagging" attack makes it possible for Eve to get full information on the key without inducing noise.

1.4 Our Contribution

We suggest a new SQKD protocol, similar to "QKD with classical Alice", that is experimentally feasible: in the original protocol of "QKD with classical Alice", Alice could choose only between two operations (CTRL and SIFT); in our new protocol, Alice may choose between four operations (CTRL, SIFT-1, SIFT-0, and SIFT-ALL), some of them (SIFT-1 and SIFT-0) corresponding to possible reflections of <u>pulses</u> by using a controllable mirror, rather than reflecting a qubit <u>as a whole</u> (CTRL).

   We can describe the new protocol in the terms of photon pulses that correspond to distinct time-bins: the state $|0\rangle$ corresponds to one photon in the first time-bin; the state $|1\rangle$ corresponds to one photon in the second time-bin; and the states $|+\rangle \triangleq \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|-\rangle \triangleq \frac{|0\rangle-|1\rangle}{\sqrt{2}}$ correspond to superpositions of pulses in the two time-bins. Therefore, $|0\rangle$ and $|1\rangle$ can be seen as classical states, while $|+\rangle$ and $|-\rangle$ are strictly quantum states. In this case, the CTRL operation corresponds to operating the mirror on both pulses (reflecting both pulses back to the originator, Bob); the SIFT-1 (SIFT-0) operation corresponds to operating the mirror only on the $|0\rangle$ ($|1\rangle$) pulse, while measuring the other pulse; and the SIFT-ALL operation corresponds to measuring all the pulses, without reflecting any of them.

   This protocol is experimentally feasible and is safe against the attack described in [20]. Moreover, we prove this protocol to be completely robust against an attacker Eve that is allowed to do anything allowed by the laws of quantum physics, including the possibility of sending multi-photon pulses (namely, assuming that Eve may use any quantum state consisting of the two modes (i.e., two qubit-states) $|0\rangle$ and $|1\rangle$).

2 Preliminaries

2.1 The Notations of Quantum Information

In quantum information, information is represented by a quantum state. A quantum pure state is denoted by $|\psi\rangle$, and is a normalized vector in a Hilbert space. The qubit Hilbert space is $\mathcal{H}_2 = \mathrm{Span}\{|0\rangle, |1\rangle\}$, with $|0\rangle$ and $|1\rangle$ being two orthonormal vectors; two other important states in $\mathcal{H}_2$ are $|+\rangle \triangleq \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|-\rangle \triangleq \frac{|0\rangle-|1\rangle}{\sqrt{2}}$.

   A quantum mixed state is a probability distribution of several pure states, and is represented by a density matrix: $\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$, where p_j is the probability that the system is in the pure state $|\psi_j\rangle$ (this definition should not be confused with the probabilities of measurement results). For example, if the mixed state of a system is $\rho_1 = \frac{1}{3}|0\rangle\langle0| + \frac{2}{3}|+\rangle\langle+|$, this means that the system is in the $|0\rangle$ state with probability $\frac{1}{3}$ and in the $|+\rangle$ state with probability $\frac{2}{3}$.

   The most general operations allowed by quantum physics for the Hilbert space $\mathcal{H}$ are: performing any unitary transformation $U : \mathcal{H} \to \mathcal{H}$; adding an ancillary state inside another Hilbert space; measuring a state with respect to some orthonormal basis; and tracing out a quantum system (namely, ignoring and forgetting a quantum system).

   See [15] for more background about quantum information.

2.2   The Fock Space Notations

The Fock space notations, that serve as an extension of the qubit states, are as follows: the Fock basis vector $|0,1\rangle$ stands for a single photon in a qubit-state $|\mathrm{o}\rangle$, and the Fock basis vector $|1,0\rangle$ stands for a single photon in a qubit-state $|1\rangle$. Naturally, the Hadamard basis qubit-states are given by the superpositions of those Fock states, so that $\frac{|0,1\rangle \pm |1,0\rangle}{\sqrt{2}}$ stand for a single photon in a qubit-state $|\pm\rangle$. The general state of this photonic qubit can then be written as $\alpha|0,1\rangle + \beta|1,0\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$.

Qubits are embedded in the 2-mode Fock space

$$\mathscr{F} = \mathrm{Span}\{\, |m_1, m_\mathrm{o}\rangle \mid m_1 \geq 0, m_\mathrm{o} \geq 0 \} \ , \tag{1}$$

where m_1 and m_o are non-negative integers. The state $|m_1, m_\mathrm{o}\rangle$ represents m_1 indistinguishable photons in the qubit-state $|1\rangle$ and m_o indistinguishable photons in the qubit-state $|\mathrm{o}\rangle$.

More formally, the linear embedding $\sigma : \mathscr{H}_2 \to \mathscr{F}$ of the qubits in the Fock space is defined by $\sigma|\mathrm{o}\rangle = |0,1\rangle$ and $\sigma|1\rangle = |1,0\rangle$, and consequently

$$\mathscr{H}_2 \simeq \sigma(\mathscr{H}_2) = \mathrm{Span}\{\, |0,1\rangle, |1,0\rangle \} \ . \tag{2}$$

In particular, the state $|0,0\rangle \in \mathscr{F}$ is used for describing absence of photons (the "vacuum state").

2.3   Robustness and Security of QKD Protocols

A QKD (or SQKD) protocol is operated by two legitimate parties, normally called Alice and Bob, that try to generate a secret key shared by them. The adversary (Eve) is computationally and technologically unlimited and can do anything allowed by the laws of nature.

Following [4], the robustness of QKD and SQKD protocols is defined as follows:

Definition 6   (Robustness of a QKD Protocol).

– *A QKD or SQKD protocol is **completely robust** if, assuming that the parties' probability of finding any error in the bits tested by the protocol equals to zero, Eve cannot obtain any information on the raw key (namely, on the bit string held by Alice and Bob before the error correction and the privacy amplification steps, that give the final key).*
– *A QKD or SQKD protocol is **completely nonrobust** if, assuming that the parties' probability of finding any error in the bits tested by the protocol equals to zero, Eve can still get full information on the raw key.*
– *A QKD or SQKD protocol is **partly robust** if, assuming that the parties' probability of finding any error in the bits tested by the protocol equals to zero, Eve can acquire some limited information on the raw key.*

In contrast, a "security" of a QKD protocol (informally) means that, except with an exponentially-small probability ε, the protocol either aborts or generates a secret key

that is the same as a perfect key: namely, it is uniformly distributed, is the same for Alice and Bob, and is independent of Eve's information [17,16].

As said in [4], a completely nonrobust protocol is automatically insecure, because Eve may steal the whole raw key, while Alice and Bob cannot notice that and will not abort the protocol. However, a completely robust protocol is not necessarily secure.

3   The "Classical Alice with a Controllable Mirror" Protocol

3.1   Description of the Protocol

The Classical Operations.

Definition 7   (Classical Operations in "Classical Alice with a Controllable Mirror").
The classical operations of Alice in the "Classical Alice with a controllable mirror" protocol, given her initial probe state $|0,0\rangle_A$ and a state sent from Bob $|m_1,m_0\rangle_B$ (both represented by using the Fock space notations), are:

I (CTRL) *Do nothing: (using the classical operation 4 described in Sect. 1.2)*

$$I|0,0\rangle_A |m_1,m_0\rangle_B = |0,0\rangle_A |m_1,m_0\rangle_B \tag{3}$$

S_1 (SIFT-1) *Swap half of Alice's probe with the $|m_1\rangle_B$ half of Bob's state: (using the classical operations 1+4 described in Sect. 1.2, each applied on one of the two pulses only)*

$$S_1|0,0\rangle_A |m_1,m_0\rangle_B = |m_1,0\rangle_A |0,m_0\rangle_B \tag{4}$$

S_0 (SIFT-0) *Swap half of Alice's probe with the $|m_0\rangle_B$ half of Bob's state: (using the classical operations 1+4 described in Sect. 1.2, each applied on one of the two pulses only)*

$$S_0|0,0\rangle_A |m_1,m_0\rangle_B = |0,m_0\rangle_A |m_1,0\rangle_B \tag{5}$$

S (SIFT-ALL) *Swap the entire probe of Alice with the entire state $|m_1,m_0\rangle_B$ of Bob: (using the classical operation 1 described in Sect. 1.2)*

$$S|0,0\rangle_A |m_1,m_0\rangle_B = |m_1,m_0\rangle_A |0,0\rangle_B \tag{6}$$

After each of those operations, Alice measures her probe (the A state) in the computational basis and sends to Bob the B state.

In the protocol, Alice's actions are described as attaching a probe in Fock space $\mathscr{F}$, applying a swap transformation, and performing a measurement in the computational basis. This description is meant to match the general framework of measurements in quantum information, and it corresponds to the operation performed by Alice: this is a good description of the usage of a mirror (with $|0\rangle$ and $|1\rangle$ being two photon pulses), such that Alice can decide whether the mirror reflects both pulses (CTRL), just the first pulse (SIFT-1), just the second pulse (SIFT-0), or none of the pulses (SIFT-ALL).

A Limitation of the Measurement Devices. In the current reliable implementations of QKD that use the current technology, Alice and Bob are limited in the sense that they cannot *count* the number of photons in each qubit-state (e.g., count how many photons they detect in the qubit-state $|0\rangle$ and how many photons they detect in the qubit-state $|1\rangle$), but can only check whether they get <u>any</u> photon in the qubit-state $|0\rangle$ or not, and also check whether they get <u>any</u> photon in the qubit-state $|1\rangle$ or not. For our protocol (and its robustness analysis) to be practical, we assume that Alice and Bob are indeed limited in that sense. Therefore, when Alice and Bob measure in the computational basis, their measurement results are denoted as $\hat{m}_1\hat{m}_0$, with $\hat{m}_0, \hat{m}_1 \in \{0,1\}$. Similarly, when Bob measures in the Hadamard basis, his measurement result is $\hat{m}_-\hat{m}_+$, with $\hat{m}_+, \hat{m}_- \in \{0,1\}$. This limitation leads to the following definition:

Definition 8 ("Count" of a Measurement Result). *Let us look at a measurement result of Alice or Bob (that is* 00, 01, 10, *or* 11*). The "count" of this measurement result is the number of distinct qubit-states detected during the measurement.*

The above definitions are summarized in Table 1.

Table 1. The four possible measurement results by Alice or Bob (measuring in the computational basis), depending on the state obtained him or her (that is represented in the Fock space notations)

Obtained State	Measurement Result	"Count"
$|0,0\rangle$	00	0
$|0,m_0\rangle$ $(m_0 > 0)$	01	1
$|m_1,0\rangle$ $(m_1 > 0)$	10	1
$|m_1,m_0\rangle$ $(m_1 > 0, m_0 > 0)$	11	2

The Protocol.

Protocol 9 ("Classical Alice with a Controllable Mirror"). *The "Classical Alice with a controllable mirror" protocol (using a two-way quantum channel), operated by the two parties Alice and Bob (where Alice is classical, in the sense defined in section 1.2), consists of the following steps:*

1. *Bob sends to Alice N quantum states, all of them equal to* $|+\rangle_B \triangleq \frac{|0\rangle_B + |1\rangle_B}{\sqrt{2}}$*; or, in the Fock space notations, to* $\sigma|+\rangle_B = \frac{|0,1\rangle_B + |1,0\rangle_B}{\sqrt{2}}$*.*
2. *For each of the received states, Alice adds a probe state* $|0,0\rangle_A$ *(so that the global state should be* $|0,0\rangle_A\sigma|+\rangle_B$*), and then randomly chooses one of the four classical operations listed in Definition 7 (CTRL, SIFT-1, SIFT-0, or SIFT-ALL). (See Table 2 for a list of the states that should be obtained by Bob after this step.)*
3. *Bob measures all the states he receives, choosing randomly for each state whether to measure it in the computational basis* $\{|0\rangle, |1\rangle\}$ *or in the Hadamard basis* $\{|+\rangle, |-\rangle\}$*.*

4. *Alice and Bob reveal (by using the classical channel) Bob's basis choices and Alice's operation choices (CTRL, SIFT-x (x $\in \{0, 1\}$), or SIFT-ALL; Alice does <u>not</u> reveal her choices between SIFT-1 and SIFT-0, that are kept by her as a secret bit string).*

5. *Alice and Bob discard the states that Bob measured in the wrong basis (namely, they discard the CTRL states he measured in the computational basis, and the SIFT-x states he measured in the Hadamard basis).*

6. *For each of the SIFT-x and SIFT-ALL states, Alice and Bob publicly announce the "count" of their measurement result (as defined in Definition 8).*

7. *Alice and Bob consider the following types of measurement results (for the non-discarded states) as errors, and estimate the error rate:*
   - *For each CTRL state (measured by Bob in the Hadamard basis): detection of a photon in the qubit-state $|-\rangle$ by Bob (namely, a measurement result of either 10 or 11)*
   - *For each SIFT-x state (measured by Bob in the computational basis): a "count" of 1 by <u>both</u> Alice and Bob; or a "count" of 2 by Bob*
   - *For each SIFT-ALL state: a "count" of 1 by Bob; or a "count" of 2 by <u>either</u> Alice or Bob*

8. *Alice and Bob consider the following types of measurement results (for the non-discarded states) as losses, to be discarded:*
   - *For each CTRL state (measured by Bob in the Hadamard basis): a loss by Bob (namely, a measurement result of 00)*
   - *For each SIFT-x state (measured by Bob in the computational basis): a "count" of 0 by <u>both</u> Alice and Bob*
   - *For each SIFT-ALL state: a "count" of 0 by Alice*

9. *Alice and Bob discard all the SIFT-x states for which Alice's "count" is 1 <u>and</u> Bob's "count" is 0, and all the SIFT-x states that had errors (in Step 7) or that were lost (in Step 8).*
   Now, for all the non-discarded SIFT-x states, Bob's "count" is 1 and Alice's "count" is 0. For each of those states, Alice and Bob share a (secret) bit b, because Alice knows (in secret) what operation S_{1-b} she performed, and Bob knows (in secret) what state $\sigma |b\rangle$ he measured. Each one of Alice and Bob keeps this sequence of bits b as his or her bit string.

10. *Alice and Bob reveal some random subset of their bit strings, compare them, and estimate the error rate (this is the error rate in the way from Alice back to Bob). They abort the protocol if the error rate in those bits, or any of the error rates measured in Step 7, is above a specified threshold. They discard the revealed bits.*

11. *Now Alice and Bob keep only the string of SIFT-x bits that were not discarded, decided as described in Step 9. If there is no noise or eavesdropping, this bit string should be the same for Alice and Bob.*

12. *Alice sends to Bob error correction information, and Bob corrects the errors in his bit string, so that it is the same as Alice's.*

13. *Alice and Bob perform a privacy amplification process, yielding a final key that is identical and is fully secure from any eavesdropper.*

Notice that Bob does not have a special role in Step 1: he always generates the same state, $|+\rangle$. For the matter of fact, it is even possible that the adversary Eve generates this state instead of him.

Table 2. The states received by Bob (depending on Alice's operation) after Step 2 of Protocol 9, if there is no noise and no eavesdropping (the states for SIFT-1 and SIFT-0 are mixed states; see Sect. 2.1 for details)

Alice's Operation	Bob's Received State
CTRL	$\sigma\,\lvert+\rangle_{\mathrm{B}}$
SIFT-1	$1/2 \cdot \left[\,\lvert 0,1\rangle_{\mathrm{B}}\langle 0,1\rvert_{\mathrm{B}} + \lvert 0,0\rangle_{\mathrm{B}}\langle 0,0\rvert_{\mathrm{B}}\right]$
SIFT-0	$1/2 \cdot \left[\,\lvert 1,0\rangle_{\mathrm{B}}\langle 1,0\rvert_{\mathrm{B}} + \lvert 0,0\rangle_{\mathrm{B}}\langle 0,0\rvert_{\mathrm{B}}\right]$
SIFT-ALL	$\lvert 0,0\rangle_{\mathrm{B}}$

3.2 Robustness Analysis

Notations and Assumptions. We notice that according to the protocol, Alice and Bob assign the following interpretations to all the measurement results (except the results of measurements discarded in Step 5 of the protocol), where "count" has been defined in Definition 8:

- For CTRL states (measured by Bob in the Hadamard basis), only Bob performs a measurement. The interpretations are detailed in Table 3.
- For SIFT-x states (measured by Bob in the computational basis), both Alice and Bob perform measurements and reveal their "counts". The interpretations are detailed in Table 4.
- For SIFT-ALL states, the interpretations are detailed in Tables 5 and 6.

Table 3. The interpretations of Bob's measurement results for CTRL states

Bob's Result	Interpretation
00	a loss
01 (i.e., $\sigma\,\lvert+\rangle$)	a legal result
10 (i.e., $\sigma\,\lvert-\rangle$)	an error
11	an error

Eve's attack on a state can be performed in both directions: from the source (Bob) to Alice, Eve applies U; from Alice back to Bob, Eve applies V. We may assume that Eve uses a fixed probe space $\mathscr{H}_{\mathrm{E}}$ for her attacks.

To prove robustness, we will prove that for Eve's attack to be undetectable by Alice and Bob (namely, for Eve's attack not to cause any errors), it must not give Eve any information.

If Alice and Bob cannot detect eavesdropping, the following conditions must be satisfied for all the measurement results that were not discarded in Step 5 of the protocol (otherwise, eavesdropping may be detected):

Table 4. The interpretations of Alice's and Bob's measurement results for SIFT-x states

Alice's "Count"	Bob's "Count"	Interpretation
0	0	a loss
0	1	Alice and Bob share a bit
1	0	Alice and Bob do not share a bit
1	1	an error
0 or 1	2	an error
2		impossible

Table 5. The interpretations of Alice's measurement results for SIFT-ALL states

Alice's Result	Interpretation
00	a loss
01 or 10	a legal result
11	an error

Table 6. The interpretations of Bob's measurement results for SIFT-ALL states

Bob's Result	Interpretation
00	a legal result
01, 10 or 11	an error

1. For every CTRL state, Bob's measurement result (in the Hadamard basis) must not be 10 or 11 (namely, Bob must never detect a $|-\rangle$ qubit).
2. For every SIFT-x state, Alice's "count" and Bob's "count" (in the computational basis) must not be both 1.
3. For every SIFT-x state, Bob's "count" (in the computational basis) must not be 2 (namely, Bob's measurement result must not be 11).
4. For every SIFT-x state, no error (that may be detected during Step 10 of the protocol) can exist. In other words:
   (a) For every SIFT-1 state, Bob's measurement result (in the computational basis) must not be 10.
   (b) For every SIFT-0 state, Bob's measurement result (in the computational basis) must not be 01.
5. For every SIFT-ALL state, Alice's measurement result must not be 11.
6. For every SIFT-ALL state, Bob's measurement result must not be 01, 10 or 11.

A Lemma Restricting Eve's Actions.

Extending the Fock Space Notations. We remember that the 2-mode Fock space is

$$\mathscr{F} = \text{Span}\{ |m_1, m_0\rangle \mid m_1 \geq 0, m_0 \geq 0 \} , \tag{7}$$

where the state $|m_1, m_0\rangle$ represents m_1 indistinguishable photons in the qubit-state $|1\rangle$ and m_0 indistinguishable photons in the qubit-state $|0\rangle$.

Similarly, Fock space can be represented as

$$\mathscr{F} = \text{Span}\{ |m_-, m_+\rangle_x \mid m_- \geq 0, m_+ \geq 0 \} , \tag{8}$$

where the state $|m_-, m_+\rangle_x$ represents m_- indistinguishable photons in the qubit-state $|-\rangle$ and m_+ indistinguishable photons in the qubit-state $|+\rangle$.

The Lemma. We now prove the following Lemma, that will later be used to restrict Eve's possible attacks:

Lemma 10. *If $|\psi'\rangle = \sum_{m>0} [|m,0\rangle_B |F_{m,0}\rangle_E + |0,m\rangle_B |G_{0,m}\rangle_E] + |0,0\rangle_B |H\rangle_E$ is a bipartite state in $\mathscr{H}_B \otimes \mathscr{H}_E$, and if there is a zero probability of measuring any basis state $|m_-, m_+\rangle_{x,B}$ of $\mathscr{H}_B$ with $m_- > 0$ (namely, there is a zero probability that Bob gets a photon in the qubit-state $|-\rangle$), then $|F_{1,0}\rangle_E = |G_{0,1}\rangle_E$, and $|F_{m,0}\rangle_E = |G_{0,m}\rangle_E = 0$ for all $m > 1$.*

Proof. For $m = 1$, since $|0,1\rangle_B = \frac{|0,1\rangle_{x,B} + |1,0\rangle_{x,B}}{\sqrt{2}}$ and $|1,0\rangle_B = \frac{|0,1\rangle_{x,B} - |1,0\rangle_{x,B}}{\sqrt{2}}$, we get

$$|1,0\rangle_B |F_{1,0}\rangle_E + |0,1\rangle_B |G_{0,1}\rangle_E \tag{9}$$
$$= \frac{|0,1\rangle_{x,B}}{\sqrt{2}} [|G_{0,1}\rangle_E + |F_{1,0}\rangle_E] + \frac{|1,0\rangle_{x,B}}{\sqrt{2}} [|G_{0,1}\rangle_E - |F_{1,0}\rangle_E] ,$$

and since the probability of getting a photon in the qubit-state $|-\rangle$ must be 0, it is necessary that $|F_{1,0}\rangle_E = |G_{0,1}\rangle_E$.

For $m > 1$, using the ladder operators a_{o}, a_{1}, a_+, and a_-, since $a_{\mathsf{o}} = \frac{a_+ + a_-}{\sqrt{2}}$ and $a_{\mathsf{1}} = \frac{a_+ - a_-}{\sqrt{2}}$, it follows that

$$|0,m\rangle_{\mathrm{B}} = \frac{a_{\mathsf{o}}^{\dagger\,m}|0,0\rangle_{\mathrm{B}}}{\sqrt{m!}} = \frac{1}{\sqrt{2^m \cdot m!}} \sum_{k=0}^{m} \binom{m}{k} a_-^{\dagger\,k} a_+^{\dagger\,m-k} |0,0\rangle_{\mathrm{B}} \tag{10}$$

$$|m,0\rangle_{\mathrm{B}} = \frac{a_{\mathsf{1}}^{\dagger\,m}|0,0\rangle_{\mathrm{B}}}{\sqrt{m!}} = \frac{1}{\sqrt{2^m \cdot m!}} \sum_{k=0}^{m} \binom{m}{k} (-1)^k a_-^{\dagger\,k} a_+^{\dagger\,m-k} |0,0\rangle_{\mathrm{B}} \ . \tag{11}$$

From (10) and (11) it follows that

$$|m,0\rangle_{\mathrm{B}} |F_{m,0}\rangle_{\mathrm{E}} + |0,m\rangle_{\mathrm{B}} |G_{0,m}\rangle_{\mathrm{E}} \tag{12}$$
$$= |e^{(m)}\rangle_{\mathrm{B}} \left[|G_{0,m}\rangle_{\mathrm{E}} + |F_{m,0}\rangle_{\mathrm{E}} \right] + |o^{(m)}\rangle_{\mathrm{B}} \left[|G_{0,m}\rangle_{\mathrm{E}} - |F_{m,0}\rangle_{\mathrm{E}} \right] \ ,$$

with

$$|e^{(m)}\rangle_{\mathrm{B}} = \frac{1}{\sqrt{2^m \cdot m!}} \sum_{k\ \mathrm{even}} \binom{m}{k} a_-^{\dagger\,k} a_+^{\dagger\,m-k} |0,0\rangle_{\mathrm{B}} \tag{13}$$

$$|o^{(m)}\rangle_{\mathrm{B}} = \frac{1}{\sqrt{2^m \cdot m!}} \sum_{k\ \mathrm{odd}} \binom{m}{k} a_-^{\dagger\,k} a_+^{\dagger\,m-k} |0,0\rangle_{\mathrm{B}} \ , \tag{14}$$

where $a_-^{\dagger\,k} a_+^{\dagger\,m-k} |0,0\rangle_{\mathrm{B}}$ is, up to a constant factor, the Fock state $|k, m-k\rangle_{\mathrm{x,B}}$. The probability of finding a photon in the qubit-state $|-\rangle$ must be zero; thus, the coefficient of $a_-^{\dagger\,k} a_+^{\dagger\,m-k} |0,0\rangle_{\mathrm{B}}$ for $k > 0$ must be zero. Substituting $|e^{(m)}\rangle_{\mathrm{B}}$ and $|o^{(m)}\rangle_{\mathrm{B}}$ by their values in (12), the coefficient of $a_-^{\dagger\,k} a_+^{\dagger\,m-k} |0,0\rangle_{\mathrm{B}}$ is (up to a non-zero constant factor) $|G_{0,m}\rangle_{\mathrm{E}} + |F_{m,0}\rangle_{\mathrm{E}}$ for even k and $|G_{0,m}\rangle_{\mathrm{E}} - |F_{m,0}\rangle_{\mathrm{E}}$ for odd k. Since $k = m > 0$ and $k = m-1 > 0$ have different parities, this implies both $|G_{0,m}\rangle_{\mathrm{E}} + |F_{m,0}\rangle_{\mathrm{E}} = 0$ and $|G_{0,m}\rangle_{\mathrm{E}} - |F_{m,0}\rangle_{\mathrm{E}} = 0$, and thus $|F_{m,0}\rangle_{\mathrm{E}} = |G_{0,m}\rangle_{\mathrm{E}} = 0$. $\square$

A Complete Robustness Proof. The following theorem proves our protocol to be completely robust:

Theorem 11. *The "Classical Alice with a controllable mirror" protocol is robust against any attack by Eve (assuming that all the 2-mode Fock space states may be used by Eve).*

Proof. We assume that during a run of the protocol, no error can be detected by Alice and Bob; and we prove that Eve gets no information on the raw key.

On the way from the source (Bob) to Alice, Eve applies a unitary transformation U on Bob's original state $\sigma |+\rangle_{\mathrm{B}}$ and on an initial probe she attaches. Since Bob's original state is always the same state, Eve always sends the same state, that has the form (including Eve's probe)

$$|\psi\rangle = \sum_{m_{\mathsf{1}}, m_{\mathsf{o}}} |m_{\mathsf{1}}, m_{\mathsf{o}}\rangle_{\mathrm{B}} |E_{m_{\mathsf{1}}, m_{\mathsf{o}}}\rangle_{\mathrm{E}} \ , \tag{15}$$

where the $|E_{i,j}\rangle_{\mathrm{E}}$ are non-normalized vectors in $\mathscr{H}_{\mathrm{E}}$. In fact, we may assume that Eve generates this state by herself (and blocks the $\sigma |+\rangle_{\mathrm{B}}$ state sent by Bob).

After Alice has attached her probe $|0,0\rangle_A$, the full state (including Alice's and Eve's probes) is

$$\sum_{m_1,m_o} |0,0\rangle_A |m_1,m_o\rangle_B |E_{m_1,m_o}\rangle_E \ . \tag{16}$$

Condition 5 means that $|E_{m_1,m_o}\rangle_E = 0$ for all m_1, m_o satisfying $m_1 > 0$ and $m_o > 0$. Therefore,

$$|\psi\rangle = |\psi_0'\rangle + |\psi_1'\rangle + |0,0\rangle_B |E_{0,0}\rangle_E \ , \tag{17}$$

with

$$|\psi_0'\rangle = \sum_{m_1>0} |m_1,0\rangle_B |E_{m_1,0}\rangle_E, \quad |\psi_1'\rangle = \sum_{m_o>0} |0,m_o\rangle_B |E_{0,m_o}\rangle_E \ . \tag{18}$$

Alice now applies one of the four possible operations (CTRL $= I$, SIFT-1 $= S_1$, SIFT-0 $= S_0$, or SIFT-ALL $= S$) and destructively measures her probe state. The (non-normalized) state of the Bob+Eve system after Alice's operation (and measurement) is written in Table 7.

Table 7. The (non-normalized) state of the Bob+Eve system after Alice's operation

Alice's Operation	Alice's "Count"	Bob+Eve State
CTRL	0	$|\psi_0'\rangle + |\psi_1'\rangle + |0,0\rangle_B |E_{0,0}\rangle_E$
SIFT-1	0	$|\psi_1\rangle = |\psi_1'\rangle + |0,0\rangle_B |E_{0,0}\rangle_E$
SIFT-0	0	$|\psi_0\rangle = |\psi_0'\rangle + |0,0\rangle_B |E_{0,0}\rangle_E$
SIFT-1	1	$\rho_1 = \sum_{m_1>0} |0,0\rangle_B \langle 0,0|_B \otimes |E_{m_1,0}\rangle_E \langle E_{m_1,0}|_E$
SIFT-0	1	$\rho_0 = \sum_{m_o>0} |0,0\rangle_B \langle 0,0|_B \otimes |E_{0,m_o}\rangle_E \langle E_{0,m_o}|_E$
SIFT-ALL		$\rho_1 + \rho_0 + |0,0\rangle_B \langle 0,0|_B \otimes |E_{0,0}\rangle_E \langle E_{0,0}|_E$

Then, Eve applies a second unitary transformation V on the state sent from Alice to Bob (and on her own probe state). According to conditions 2, 3, and 6, it must hold that the density matrices $V\rho_1 V^\dagger$, $V\rho_0 V^\dagger$, and $V(\rho_1 + \rho_0 + |0,0\rangle_B \langle 0,0|_B \otimes |E_{0,0}\rangle_E \langle E_{0,0}|_E) V^\dagger$ only overlap with $|0,0\rangle_B$. It follows that there exists $|H_{0,0}\rangle_E \in \mathscr{H}_E$ such that:

$$V |0,0\rangle_B |E_{0,0}\rangle_E = |0,0\rangle_B |H_{0,0}\rangle_E \tag{19}$$

Let $V |\psi_0'\rangle = \sum_{m_1,m_o} |m_1,m_o\rangle_B |F_{m_1,m_o}\rangle_E$. Let us look at a SIFT-0 state for which Alice's "count" is 0. For this state, the Bob+Eve state after Eve's attack is

$$V |\psi_0\rangle = V |\psi_0'\rangle + V |0,0\rangle_B |E_{0,0}\rangle_E \tag{20}$$
$$= \sum_{m_1,m_o} |m_1,m_o\rangle_B |F_{m_1,m_o}\rangle_E + |0,0\rangle_B |H_{0,0}\rangle_E \ ,$$

and following conditions 4b and 3, Bob must not detect a photon in the qubit-state $|o\rangle$ (otherwise, the error may be detected during Step 10 of the protocol). Therefore, $|F_{m_1,m_o}\rangle_E = 0$ for all $m_o > 0$. It follows that if we denote $|H_{1,0}\rangle_E \triangleq |F_{0,0}\rangle_E$, then

$$V |\psi_0'\rangle = \sum_{m_1>0} |m_1,0\rangle_B |F_{m_1,0}\rangle_E + |0,0\rangle_B |H_{1,0}\rangle_E \ . \tag{21}$$

Similarly (following conditions 4a and 3),

$$V|\psi_1'\rangle = \sum_{m_\circ > 0} |0, m_\circ\rangle_B |G_{0,m_\circ}\rangle_E + |0, 0\rangle_B |H_{0,1}\rangle_E \ . \tag{22}$$

Now, (19), (21) and (22) imply that if Alice applies CTRL, the Bob+Eve state after Eve's attack is

$$\sum_{m > 0} \left[|m, 0\rangle_B |F_{m,0}\rangle_E + |0, m\rangle_B |G_{0,m}\rangle_E \right] + |0, 0\rangle_B |H\rangle_E \tag{23}$$

with $|H\rangle_E = |H_{1,0}\rangle_E + |H_{0,1}\rangle_E + |H_{0,0}\rangle_E$. Following condition 1, the probability of Bob getting $|m_-, m_+\rangle_x$ for $m_- > 0$ must be 0. Applying Lemma 10, we deduce that $|F_{m,0}\rangle_E = |G_{0,m}\rangle_E = 0$ for all $m > 1$, and that $|F_{1,0}\rangle_E = |G_{0,1}\rangle_E \triangleq |F\rangle_E$.

It follows that the Bob+Eve states after Eve's attack, when Alice's "count" on SIFT-1 and SIFT-0 is 0 (those are the only states for which Alice and Bob may share a bit), are

$$V|\psi_1\rangle = |0, 1\rangle_B |F\rangle_E + |0, 0\rangle_B \left[|H_{0,1}\rangle_E + |H_{0,0}\rangle_E \right] \tag{24}$$

$$V|\psi_0\rangle = |1, 0\rangle_B |F\rangle_E + |0, 0\rangle_B \left[|H_{1,0}\rangle_E + |H_{0,0}\rangle_E \right] \ . \tag{25}$$

Therefore, the state of Eve's probe is independent of all of Alice's and Bob's shared bits, and is always equal to $|F\rangle_E$ whenever Alice and Bob share a bit. Eve can thus get no information on their shared bits without being detected.   □

4 Conclusion

We have presented a new semi-quantum key distribution protocol, and have proved it robust (security analysis is left for the future). Unlike the "QKD with classical Alice" protocol, our new protocol can be experimentally implemented in a secure way, thus avoiding some attacks possible against other SQKD protocols.

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